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DEVELOPMENT OF INITIAL PERTURBATIONS OF THE EXTERNAL BOUNDARY OF AN EXPANDING GAS – LIQUID RING

S. V. Stebnovskii

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In studies of surface phenomena related to underwater explosions, in particular, in studying the process of splash dome formation, the development of perturbations in the initial stage of free surface motion is of interest. A convenient model to use in such studies is that of the flow occurring upon explosion of a cylindrical charge in a cylindrical liquid ring, where the free surface form coincides with that of the charge. The stability of an expanding liquid ring has been considered in a number of studies.

Thus, assuming an ideal incompressible liquid, [1] considered the stability of initial perturbations of a thin liquid ring expanding inertially. It was shown that in the general case such motion is unstable; introduction of surface tension has a stabilizing effect on harmonics. But in the case where the liquid motion takes place under the stimulus of impulse loading, commencement of liquid motion is preceded by exit of a shock wave onto the liquid surface, as a result of which the reflected unloading wave destroys the continuity of the liquid medium. Thus in this case the validity of using stability estimates obtained in problems concerning expansion of a continuous liquid ring is questionable.

The present study is an experimental investigation of the development of initial perturbations on the external surface of an expanding gas–liquid ring. Such a flow was realized in the following manner. Along the

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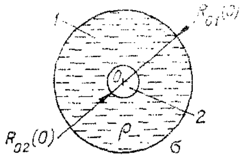

Fig. 1

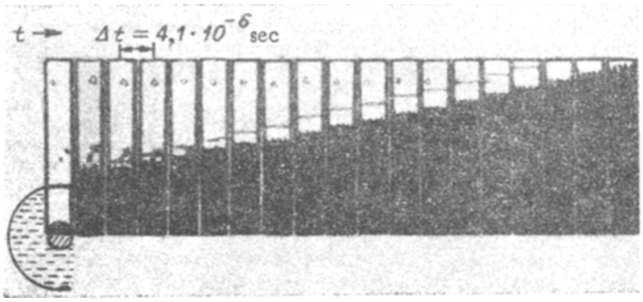

Fig. 2

axis of symmetry of a cylindrical liquid volume 1 (Fig. 1) a cylindrical charge 2 was placed (electrical detonator EDV-1); the charge radius $R_{02}(0) = 0.35$ cm; the initial radius of the external surface of the liquid cylinder $R_{01}(0)$ was varied over the range $3 \leq m = R_{01}(0)/R_{02}(0) \leq 10$; ρ is the liquid density and σ its surface tension coefficient. The liquids used were water and glycerine. The initial length of the liquid cylinder $H = 3$ cm. The cylinder ended in rigidly mounted massive steel disks 7 cm in radius. The outer cylinder surface was formed by a thin paper shell.

The initial stage of liquid expansion was recorded with an SFR-1 high-speed photorecorder. For this purpose the steel face disks confining the cylindrical volume were provided with transparent slit windows. Figure 2 shows the initial stage of the process. It is evident that immediately after charge explosion and shock wave exit to the free surface the liquid medium becomes opaque because its structure is disrupted – continuity is destroyed behind the front of the converging cylindrical unloading wave. In the very first stage of expansion of the gas–liquid layer thus formed on the free surface one can see a growth in initial perturbations. The continuation of this process was recorded with a Pentazet-16 motion-picture camera.

Thus, expansion of the gas–liquid layer is caused by the initial velocity imparted to the liquid after exit onto its surface of the shock wave, and also by expansion of the explosion bubble; braking of the layer occurs due to interaction with the surrounding air medium.

In Fig. 3a-f a graph of expanding gas–liquid ring radius versus time is shown, together with photographs of the developing perturbations on the external surface, for the case of a water cylinder $m = 5$. The exposures b-f correspond to times $t = 0$, $0.5 \cdot 10^{-3}$, 10^{-3}, $2 \cdot 10^{-3}$, $3 \cdot 10^{-3}$ sec. The second frame shows an enlargement of a portion of the gas–liquid layer in which high-frequency perturbations can be seen. It is evident from the photographs that initially when the layer expansion rate is very high, perturbations develop on its surface over a wide frequency range. But then the expansion rate falls off rapidly and the development of high-frequency perturbations is suppressed. The character of perturbation development is similar in other cases. Figure 4 shows the radius of the gas–liquid layer as a function of time: 1, glycerine cylinder $m = 10$; 2, water cylinder $m = 10$, 3, glycerine cylinder $m = 3$; 4, water cylinder $m = 3$. Figure 5 shows the corresponding motion-picture frames of perturbation development on the surface of the expanding gas–liquid layers: a, c, glycerine; b, d, water. It follows from analysis of the curves and photographs that the larger the value of m, the slower the expansion rate ralls off and the weaker the damping of high-frequency perturbations. If we compare the perturbation development of water and glycerine at identical m values, the water shows a higher frequency character.

Since analysis of the photographs and curves shows that with decrease in layer expansion rate the frequency of the perturbations which develop decreases, and moreover, it is known that surface tension is always a stabilizing factor in perturbation development, it will be of interest to analyze the behavior of perturbations as a function of the dimensionless parameter $W = \omega(t)\dot{R}_{01}^2(t)/[\sigma/R_{01}(t)]$, which characterizes the ratio of inertial to capillary forces. Here the mean (over volume) density of the gas–liquid layer $\omega(t)$ is defined as the ratio of the mass of the original cylindrical liquid layer to the current volume of the expanding gas–liquid ring $\Omega(t)$ and the density of the air component ρ_a: $\omega(t) \approx \rho\pi[R_{01}^2(0) - R_{02}^2(0) \times H/\Omega(t) + \rho_a$. The value of $\Omega(t)$ is measured ex-

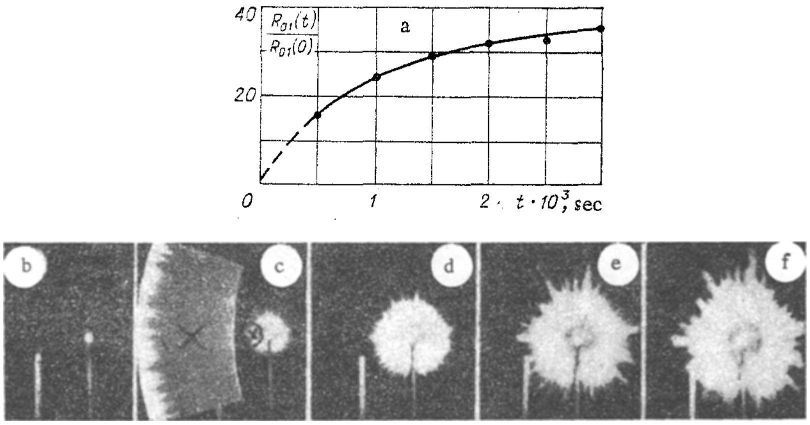

Fig. 3

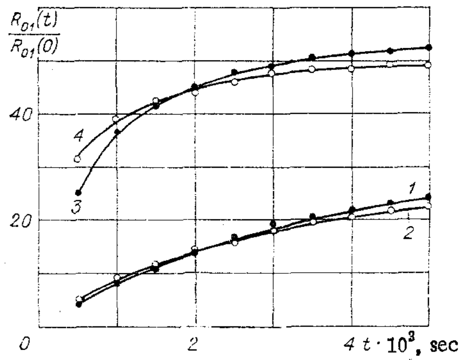

Fig. 4

perimentally from the motion-picture frames, in which the expansion process was photographed in two projections. Each frame of Fig. 5 shows the value of W(t) calculated in this manner. It is evident from Fig. 5 that the parameter W(t) does in fact characterize the behavior of the perturbations: As it decreases the frequency of perturbations on the outer surface of the gas–liquid layer decreases; the highest rate of decrease of W(t) with time occurs in the air–water layer with m = 3, and this case shows the most rapid damping of high-frequency perturbations; the lowest rate of decrease of W(t) occurs in the air–glycerine layer with m = 10, and in this case the perturbations of the ring surface show the highest frequencies

The parameter W(t) can be written in the form

$$W(t)=\frac{\omega(t)\,\dot{R}_{01}^{2}(t)}{\sigma/R_{01}(t)}=\frac{4\rho\dot{R}_{01}^{2}(0)/2}{2\sigma/R_{01}(0)}\,\frac{2\pi R_{01}(t)\,\omega(t)\,\dot{R}_{01}^{2}(t)/2}{2\pi R_{01}(0)\,\rho\dot{R}_{01}^{2}(0)/2}=4\mathrm{We}\frac{E(t)}{E(0)}=4\mathrm{We}\overline{E}(t), \tag{1}$$

where $\mathrm{We}=[\rho\dot{R}_{01}^{2}(0)/2]/[2\sigma/R_{01}(0)]$ is the Weber number, characterizing the ratio of the specific kinetic energy in the liquid ring to the free surface energy at the initial moment; E(t) is the kinetic energy of a surface layer of unit thickness with a mean density over volume of ω. Thus, the character of development of the initial perturbations of the outer surface of the expanding gas–liquid layer is defined by the ratio of inertial and capillary forces.

It is of interest to compare the mechanisms of initial perturbation development on the outer surfaces of expanding gas–liquid rings and rings of an ideal incompressible liquid. For this purpose an analytical evaluation was made of the upper frequency limit of exponentially increasing perturbations of an expanding liquid ring in the initial time period.

In a polar coordinate system r, φ, fixed to the point (Fig. 1), at time t = 0 on the radii of the inner and outer boundaries of the liquid ring $R_{0i}(0)$ there are imposed perturbations $\xi_i(0, \varphi)$, such that $|\xi_i(0, \varphi)| \ll R_{0i}(0)$, i = 1, 2. Expansion of the liquid ring is produced by both the pressure difference $p_1 - p_2$, where p_1 is the atmospheric pressure on the outer ring surface, and $p_2 = p_2(R_{02}(t))$ is the pressure on the inner surface, and by the initial mass velocity $u(0, r)u(0, R_{01}(0)) = \dot{R}_{01}(0)$).

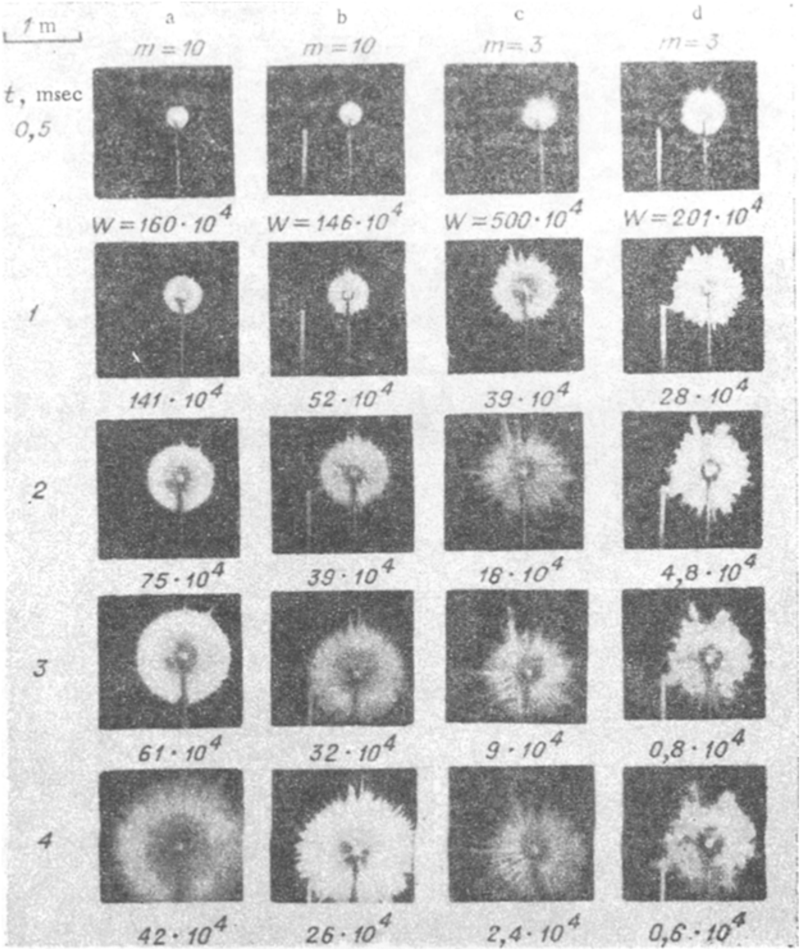

Fig. 5

For such a flow the potential boundary problem with free mobile boundaries without consideration of gravity can be written in the linear approximation as

$$\Delta\Phi(t,\ r,\ \varphi) = 0; \tag{2}$$

$$\left.\frac{\partial\Phi}{\partial r}\right|_{r=R_1} \approx \dot{R}_1,\quad \left.\frac{\partial\Phi}{\partial r}\right|_{r=R_2} \approx \dot{R}_2; \tag{3}$$

$$\left(\frac{\partial\Phi}{\partial t}\right)_{r=R_2} - \left(\frac{\partial\Phi}{\partial t}\right)_{r=R_1} + \frac{1}{2}\left[\left(\frac{\partial\Phi}{\partial r}\right)^2_{r=R_2} - \left(\frac{\partial\Phi}{\partial r}\right)^2_{r=R_1}\right] + \frac{p_2^* - p_1^*}{\rho} = 0; \tag{4}$$

$$R_i(t,\ \varphi) = R_{0i}(t) + \xi_i(t,\ \varphi),\quad i = 1,\ 2, \tag{5}$$

where $\Phi(t,\ r,\ \varphi)$ is the velocity potential; R_i is the radius of the perturbed ring boundary; $p_i^* = p_i + p_{i\sigma}$ (according to the Laplace formula, $p_{i\sigma} = \sigma/R_{0i} - \sigma\,[\xi_i + \partial^2\xi_i/\partial\varphi^2]/R_{0i}^2$).

Since Laplace equation (2) in the ring permits separation of the variables, by writing Eq. (5) in the form

$$R_1(t,\ \varphi) = R_{01}(t) + b_n(t)\cos n\varphi,\ R_2(t,\ \varphi) = R_{02}(t) + a_n(t)\cos n\varphi,\ n = 1,\ 2,\ 3, \ldots, \tag{6}$$

we obtain from Eqs. (6), (3), (2)

$$\Phi(t,\ r,\ \varphi) = \dot{R}_{0i}R_{0i}\ln r + (\delta\Phi)_1 + (\delta\Phi)_2, \tag{7}$$

$$(\delta\Phi)_1 = \frac{\frac{\dot{R}_{01}}{R_{01}}\,b_n + \dot{b}_n}{nR_{01}^{n-1}}\,r^n\cos n\varphi,\quad (\delta\Phi)_2 = \frac{\frac{\dot{R}_{02}}{R_{01}}\,a_n + \dot{a}_n}{nr^n}\,R_{02}^{n+1}\cos n\varphi.$$

The condition of independence of the perturbations of the inner and outer boundaries $((\delta\varphi)_1|_{r\to R_2}\to 0,\ (\delta\Phi)_2|_{r\to R_1}\to 0)$ is satisfied if at least $R_{02}(t)/R_{01}(t) < 0.3$, $n > 3$. Then with consideration of these limitations, imposing initial perturbations on the outer and inner ring boundaries: $\Phi = \dot{R}_{01}R_{01}\ln r + (\delta\Phi)_1$, $\Phi = \dot{R}_{02}R_{02}\ln r + (\delta\Phi)_2$, substituting these expressions in Eq. (4), with consideration of the orthogonality of the functions cos $n\varphi$ on $[0, \pi]$, after transformations we obtain

$$\ddot{R}_{01}\ln\varepsilon_1 + \frac{\dot{R}_{01}^2}{R_{01}}\left[\ln\varepsilon_1 + \left(1 - \frac{1}{\varepsilon_1}\right)\right] = -\frac{2(p_2 - p_1)}{\rho R_{01}} + \frac{2\sigma}{\rho R_{01}^2}\left(1 - \varepsilon_1^{-1/2}\right); \tag{8}$$

$$f_1\ddot{b}_n + q_1\dot{b}_n + h_1 b_n = 0; \tag{9}$$

$$\ddot{R}_{02}\ln\varepsilon_2 + \frac{\dot{R}_{02}^2}{R_{02}}\left[\ln\varepsilon_2 - \left(1 - \frac{1}{\varepsilon_2}\right)\right] = \frac{2(p_2 - p_1)}{\rho R_{02}} - \frac{2\sigma}{\rho R_{02}^2}\left(1 + \varepsilon_2^{-1/2}\right); \tag{10}$$

$$f_2\ddot{a}_n + q_2\dot{a}_n + h_2 a_n = 0, \tag{11}$$

where

$$\varepsilon_1 = \frac{R_{01}^2 - a}{R_{01}^2}; \quad \varepsilon_2 = \frac{R_{02}^2 + a}{R_{02}^2}; \quad a = R_{01}^2(0) - R_{02}^2(0); \quad p_2 = p_2(R_{02}(t));$$

$$f_1 = R_{01}(t); \; q_1 = 2\dot{R}_{01}(t); \; h_1 = \ddot{R}_{01} - n\frac{\dot{R}_{01}^2}{R_{01}} + \frac{(n^2-1)}{\rho R_{01}^2}n\sigma; \; f_2 = R_{02}(t); \; q_2 = 2\dot{R}_{02}(t); \; h_2 = \ddot{R}_{02} - n\frac{\dot{R}_{02}^2}{R_{02}} + \frac{(n^2-1)}{\rho R_{02}^2}n\sigma.$$

Equations (8), (10) describe the motion of the unperturbed outer and inner ring boundaries, while Eqs. (9), (11) describe the amplitude of the perturbations on the outer and inner boundaries, respectively. For a qualitative analysis of the behavior of perturbation amplitude, we substitute in Eqs. (9) and (11) $b_n(t) = Z_n(t) \cdot R_{01}(0)/R_{01}(t)$ and $a_n(t) = U_n(t) R_{02}(0)/R_{02}t)$, obtaining

$$\ddot{Z}_n + I_1 Z_n = 0 \text{ and } \ddot{U}_n + I_2 U_n = 0.$$

The invariants of these equations have the form

$$I_1 = -\frac{n}{R_{01}^2}\left[\dot{R}_{01}^2 - \frac{(n^2-1)}{\rho R_{01}}\sigma\right], \quad I_2 = \frac{n}{R_{02}^2}\left[\dot{R}_{02}^2 + \frac{(n^2-1)}{\rho R_{02}}\sigma\right],$$

where $\dot{R}_{01}^2(t)$, $\dot{R}_{02}^2(t)$ can be obtained by integrating Eqs. (8), (10). It follows from well-known theory of equations that at $I_2 > 0$, $a_n(t)$ oscillates during ring expansion, while $b_n(t)$ increases exponentially until $I_1 < 0$, i.e., $\dot{R}_{01}^2 > (n^2-1)\sigma/(\rho R_{01})$. From the latter inequality it follows that since n characterizes the frequency of the perturbations, the upper frequency limit of the spectrum of exponentially increasing perturbations has the form

$$n < \sqrt{1 + \frac{\rho R_{01}\dot{R}_{01}^2}{\sigma}} = \sqrt{1 + 4\frac{\rho\dot{R}_{01}^2(0)/2}{2\sigma/R_{01}(0)}\frac{2\pi R_{01}(t)\rho\dot{R}_{01}^2(t)/2}{2\pi R_{01}(0)\rho\dot{R}_{01}^2(0)/2}} \approx \sqrt{4\mathrm{We}\frac{E(t)}{E(0)}} = \sqrt{4\mathrm{We}\,\bar{E}(t)}, \tag{12}$$

where $\bar{E}(t)$ is the dimensionless kinetic energy of a surface layer of liquid of unit thickness.

Criterion (12) may be rewritten in the form $n < n^* = (4\mathrm{We}\bar{E}(t))^{1/2}$, where n^* is the upper frequency limit of exponentially developing initial perturbations on the outer surface of a ring of an ideal incompressible liquid. The right side of this inequality coincides to the accuracy of the density of the medium and the exponent with the empirical parameter $W(t)$, defining the frequency characteristic of the perturbations developing on the outer surface of an expanding gas–liquid ring.

Thus, in both cases the frequency characteristics of the developing initial perturbations depend on the product of the initial Weber number value for the expanding ring and the current value of the specific kinetic energy of a ring surface layer of unit thickness.

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